الجامعة السورية الخاصة
SYRIAN PRIVATE UNIVERSITY

كـلـيــة هندسة الحاسوب والمعلوماتية
Computer and Informatics Engineering Faculty

Electric Circuits I

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Chapter 9
Magnetic Circuits

9.1 Introduction

- Magnetism is an integral part of almost every electrical device used today in industry, research, or the home.
- Generators, motors, transformers, circuit breakers, televisions, computers, tape recorders and telephones all employ magnetic effects to perform a variety of important tasks.

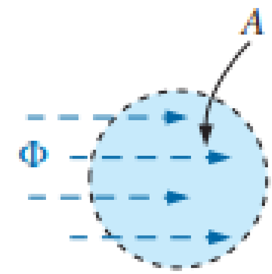
9.2 Magnetic Field

1. Flux and Flux Density

- In the Standard International (SI) system of units, **magnetic flux (ϕ)** is measured in ***webers*** (Wb).

$$1\,\text{Wb} = 10^8 \text{ lines} \Rightarrow 1\,\mu\text{Wb} = 10^2 \text{ lines}$$

- The number of flux lines per unit area is called **flux density (*B*),** and is measured in ***teslas*** (T).
  - Its magnitude is determined by the following equation: $B = \dfrac{\phi}{A}$
  - B = Wb/m^2 = teslas (T)
  - ϕ = webers (Wb)
  - A = m^2 (cross-sectional area of the magnetic field)



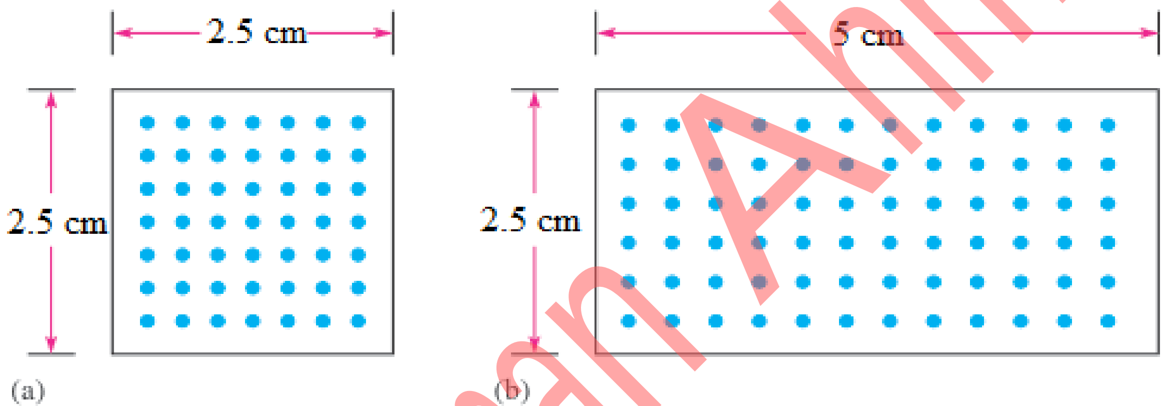
2. Permeability (μ) (النفاذية)

- Magnetic materials, such as iron, nickel, steel and alloys of these materials, have permeability hundreds and even thousands of times that of free space and are referred to as **ferromagnetic.**
- The ratio of the permeability of a material to that of free space (vacuum) is called **relative permeability (μ_r)** (النفاذية النسبية).

$$\mu_r = \frac{\mu}{\mu_0}, \qquad \mu_0 = 4\pi \times 10^{-7}\ \mathrm{Wb/At.m}$$

At.m –Ampere turns per meter.

Example 9.1. Calculate the flux and the flux density in the two magnetic cores shown in Fig. The diagram represents the cross section of a magnetized material. Assume that each dot represents 100 lines or 1 μWb.



Solution

- The flux is simply the number of lines.
- In Fig.4(a) there are 49 dots. Each represents 1 μWb , so ϕ = 49 μWb.

$$A = 0.025\,\text{m} \times 0.025\,\text{m} = 6.25 \times 10^{-4}\,\text{m}^2 \Rightarrow B = \frac{\phi}{A} = 78.4 \times 10^{-3}\,\text{Wb/m}^2 = 78.4 \times 10^{-3}\,\text{T}$$

- In Fig.4(b) there are 72 dots, so ϕ = 72 μWb.

$$A = 0.025\,\text{m} \times 0.050\,\text{m} = 1.25 \times 10^{-3}\,\text{m}^2 \Rightarrow B = \frac{\phi}{A} = 57.6 \times 10^{-3}\,\text{Wb/m}^2 = 57.6 \times 10^{-3}\,\text{T}$$

Example 9.2. If the flux density in a certain magnetic material is 0.23 T and the area of the material is 0.38 in.^{2} (inch) , what is the flux through the material?

Solution

$$39.37\,\text{in.} = 1\,\text{m} \Rightarrow A = 0.38\,\text{in.}^2\left[1\,\text{m}^2/(39.37\,\text{in.})^2\right] = 245\times10^{-6}\,\text{m}^2$$

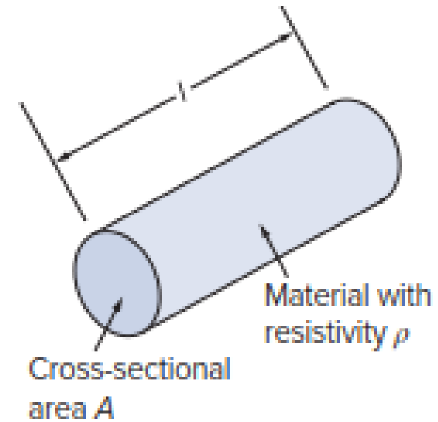
$$\phi = B\times A = (0.23\text{T})(245\times10^{-6}\,\text{m}^2) = 56.4\,\mu\,\text{Wb}$$

9.3 Reluctance

- The resistance of a material to the flow of charge (current) is determined for electric circuits by the equation

$$R = \rho \frac{l}{A}, \quad \text{(ohms}, \Omega)$$

ρ (rho) –**resistivity** of material, Ω.m



- The **reluctance** of a material to the setting up of magnetic flux lines in a material is determined by the following equation

$$\Re = \frac{l}{\mu A}, \quad \text{(At/ Wb)}$$

l = length of the path
μ = permeability (Wb/A.t m).
A = area in m^2

Magnetomotive force (mmf) (القوة المحركة المعناطيسية)

- The *cause* of magnetic flux is called magnetomotive force (mmf), which is related to the current (***I***)and number of turns (***N***) of the coil. $F_m = NI$

F_m = magnetomotive force (A.t)
N = number of turns of wire in a coil
I = current (A)

9.4 Ohm's law for electromagnetic circuits

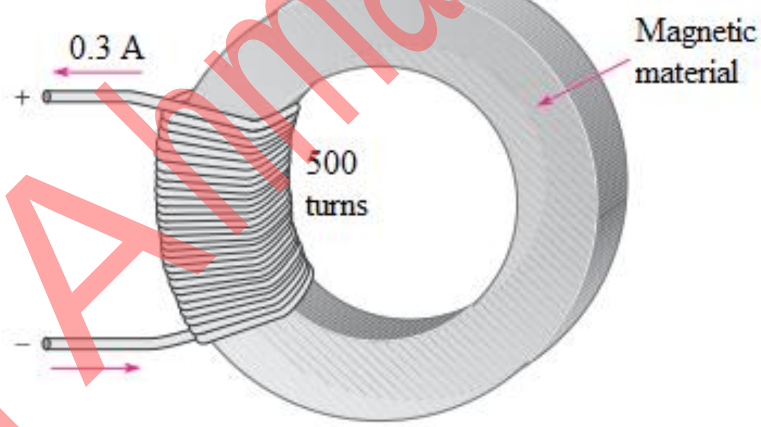
- Ohm's law for magnetic circuits is defined as

$$\phi = \frac{F_m}{\Re} = N\frac{I}{\Re}$$

- flux (ϕ) is analogous (مماثل = شبيه) to current
- magnetomotive force (F_m) is analogous to voltage
- reluctance ($\boldsymbol{R}$) is analogous to resistance.

Example 9.3. How much flux is established in the magnetic path of Fig. if the reluctance of the material is $2.8\times10^5\ \text{At/Wb}$?

Solution



$$\phi = \frac{F_m}{\Re} = \frac{NI}{\Re} = \frac{(500\,\text{t})(0.3\,\text{A})}{2.8\times10^5\ \text{At/Wb}}$$

$$= 5.36\times10^{-4}\ \mu\text{Wb}$$

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Example 9.4. There is 0.1 ampere of current through a coil with 400 turns.

a) What is the mmf?

b) What is the reluctance of the circuit if the flux is 250 mWb?

Solution

$$a)\quad N = 400\ \text{t},\ I = 0.1\ \text{A} \Rightarrow F_m = NI = 40\ \text{At}$$

$$b)\quad \Re = \frac{F_m}{\phi} = \frac{40\ \text{At}}{250\ \mu\text{Wb}} = 1.60\times10^5\ \text{At/Wb}$$

9.5 Magnetizing Force (القوة الممغنطة)

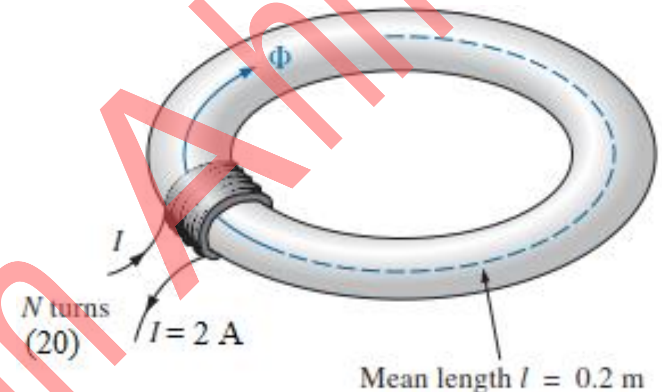
- The magnetomotive force per unit length is called the **magnetizing force (H)** or **Magnetic Field Intensity (*H*)** (شدة المجال المغناطيسي).

$$H = \frac{F_m}{l} = N\frac{I}{l}, \quad (\text{At/ m})$$

- Magnetizing force is independent of the type of core material.
- Magnetizing force is determined solely by the number of turns, the current and the length of the core.

Example 9.5.

Determine the magnetomotive force for the magnetic circuit in Fig.



Solution

$$H = \frac{NI}{l} = \frac{(20 \times 2)\text{At}}{0.2\,\text{m}} = 200\,\text{At/m}$$

- The flux density and the magnetizing force are related by the following equation:

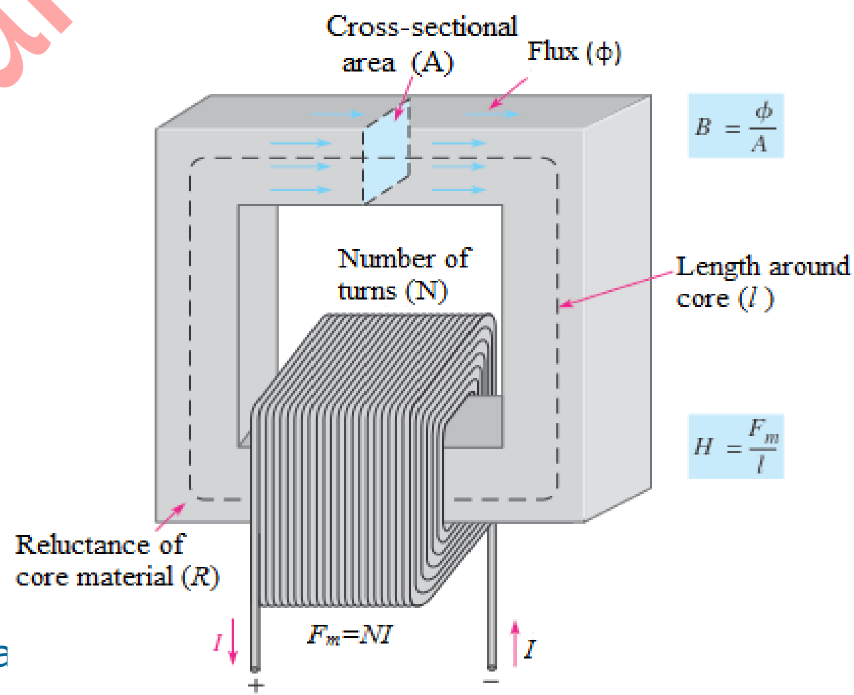
$$B = \mu H$$

$\boldsymbol{\mu}$ = permeability (Wb/A-t m).

$\boldsymbol{H}$= Magnetic field intensity (Wb/A-t m)

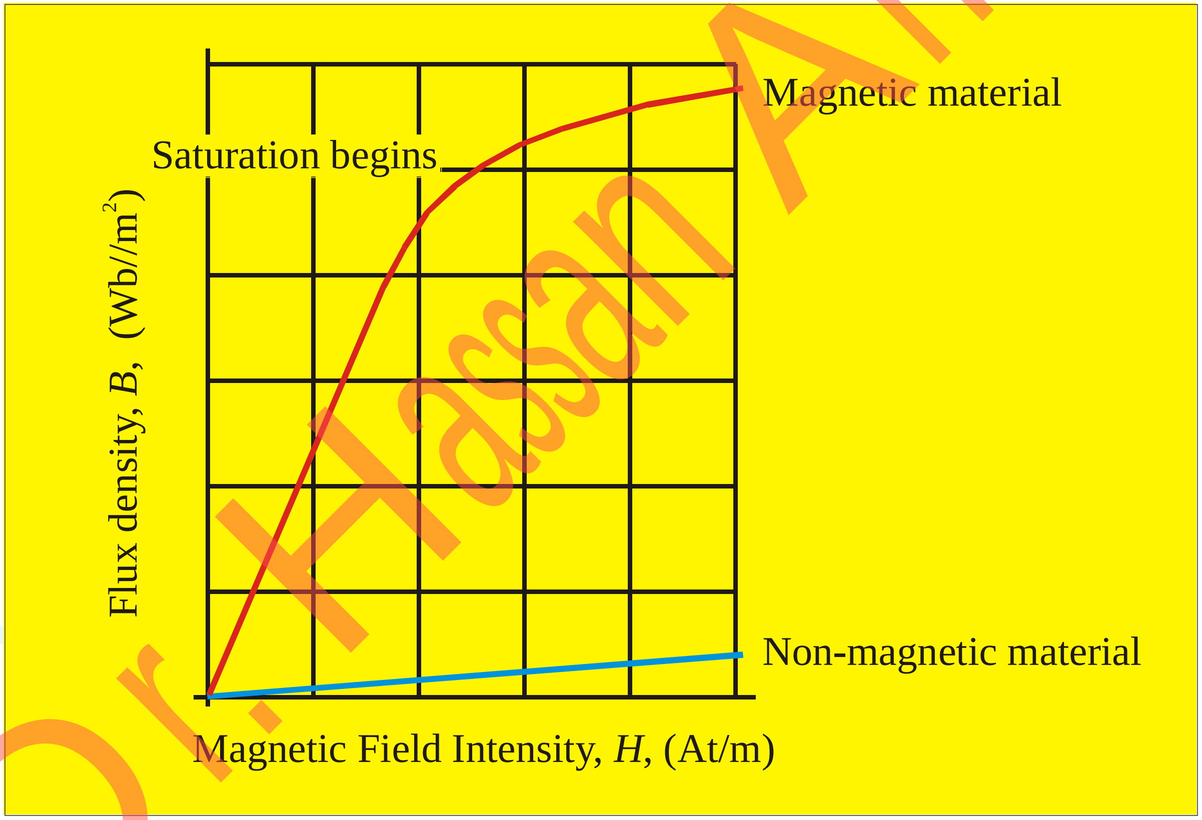
- The curve showing how these two quantities (***B*** and ***H***) are related is called the **B-H curve**, or the **hysteresis curve (منحني التخلف أو التباطؤ)**.

- The parameters that influence (المؤثرة = ذات النفوذ) both *B* and *H* are illustrated in Fig.

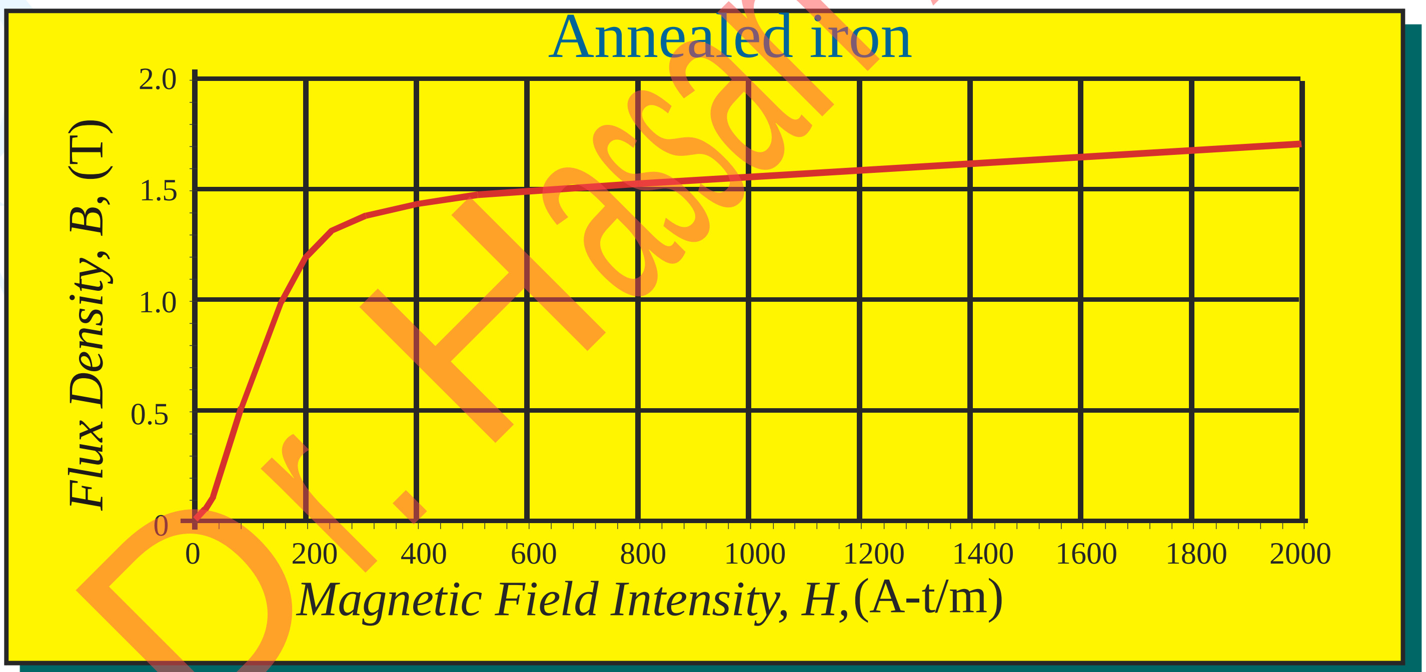


The Hysteresis Curve

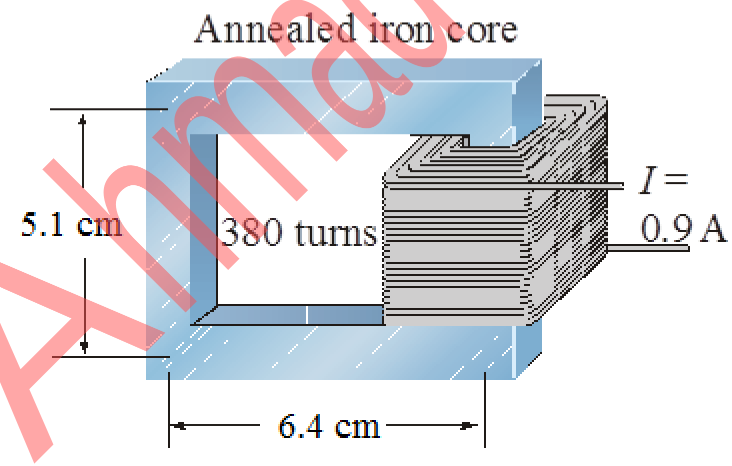
- As the graph shows, the *flux density* depends on both the *material* and the *magnetic field intensity*.



- A ***B-H* curve** is referred to as a **magnetization curve** for the case where the material is initially unmagnetized.
- A ***B-H* curve** can be *read to determine the flux density* in a given annealed iron core (قلب من الحديد الصلب).



Example 9.6. What is B for the core?

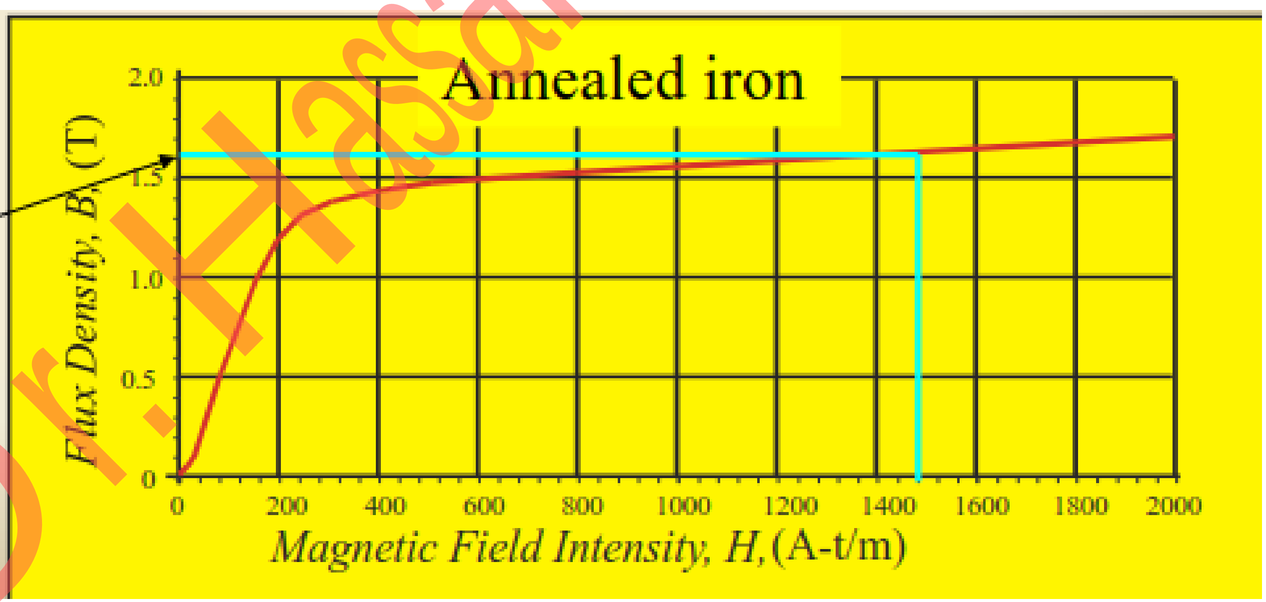


Solution

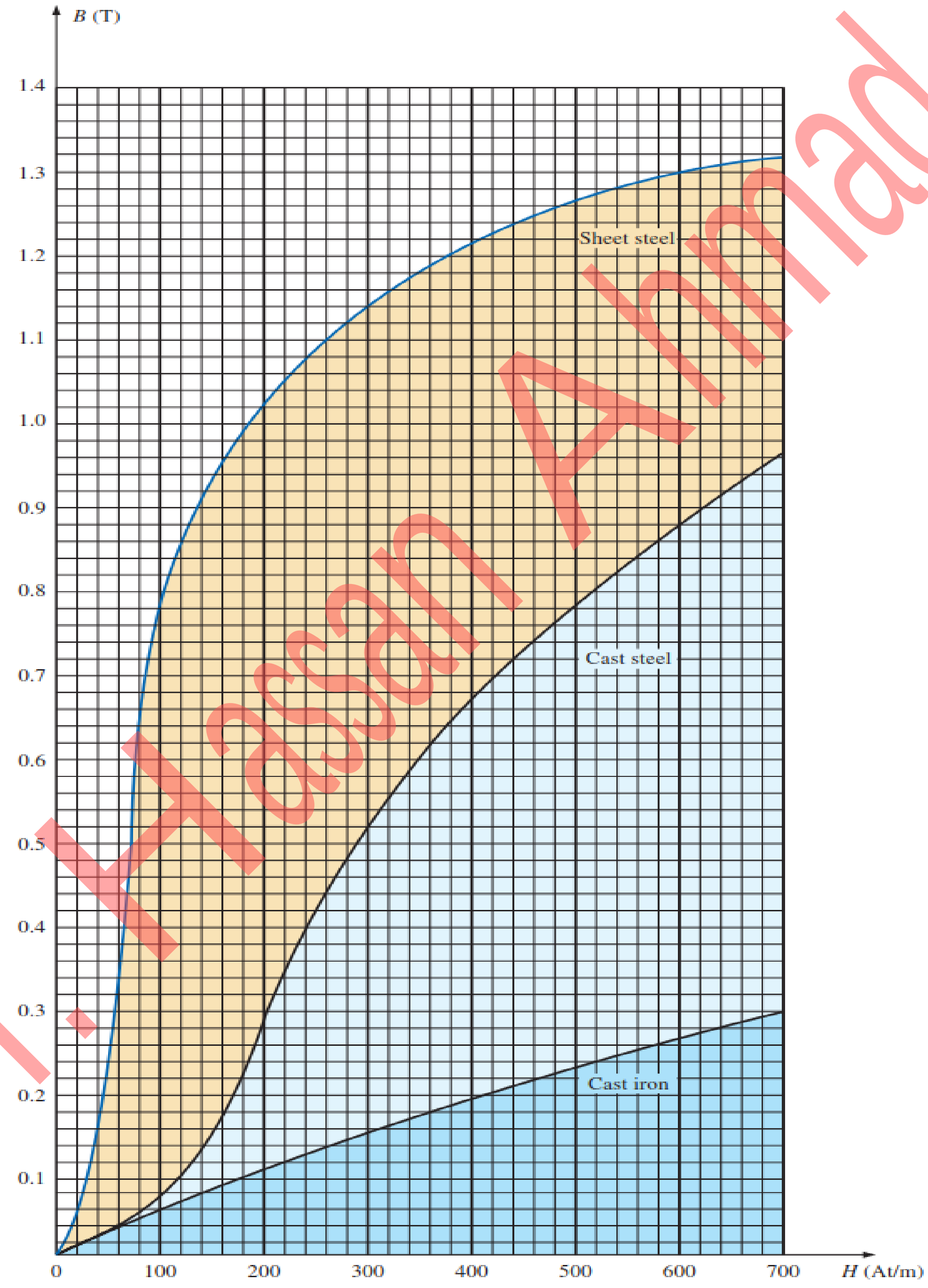
$$H = \frac{NI}{l} = \frac{(380\,\text{t})(0.9\,\text{A})}{0.23\,\text{m}} = 1487\,\text{At/m}$$

Reading the graph,

$B = 1.63$ T



The B-H curves for varies metals



9.6 Ampère's Circuital Law

Ampère's circuital law: The algebraic sum of the rises and drops of the **mmf** around a closed loop of a magnetic circuit is equal to zero; that is, the sum of the rises in mmf equals the sum drops in mmf around a closed loop.

$$\sum F_m = 0$$

$$F_m = NI;$$

$$\phi = \frac{F_m}{\Re} \Rightarrow F_m = \phi \Re$$

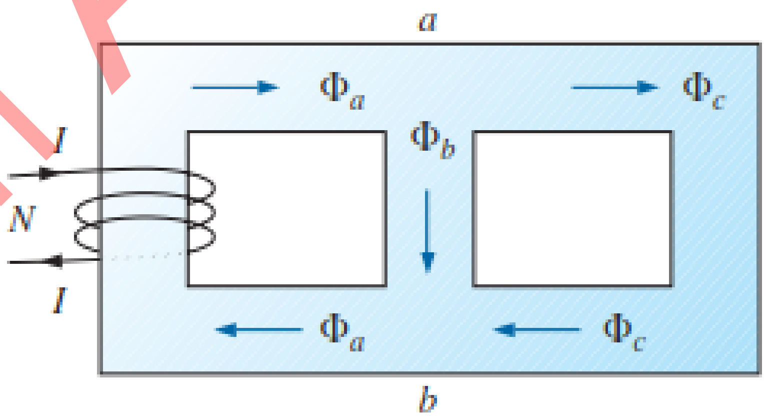
$$H = \frac{F_m}{l} \Rightarrow F_m = H\,l$$

$$NI = H\,l$$

9.7 Flux ϕ and Kirchhoff's current law

- Applying the Kirchhoff's current law, we find that the sum of the fluxes entering a junction is equal to the sum of the fluxes leaving a junction; that is,

$$\phi_a = \phi_b + \phi_c \qquad \text{(at junction } a\text{)}$$
$$\phi_b + \phi_c = \phi_a \qquad \text{(at junction } b\text{)}$$



9.8 Series Magnetic Circuits: Determining *NI*

Two types of problems:

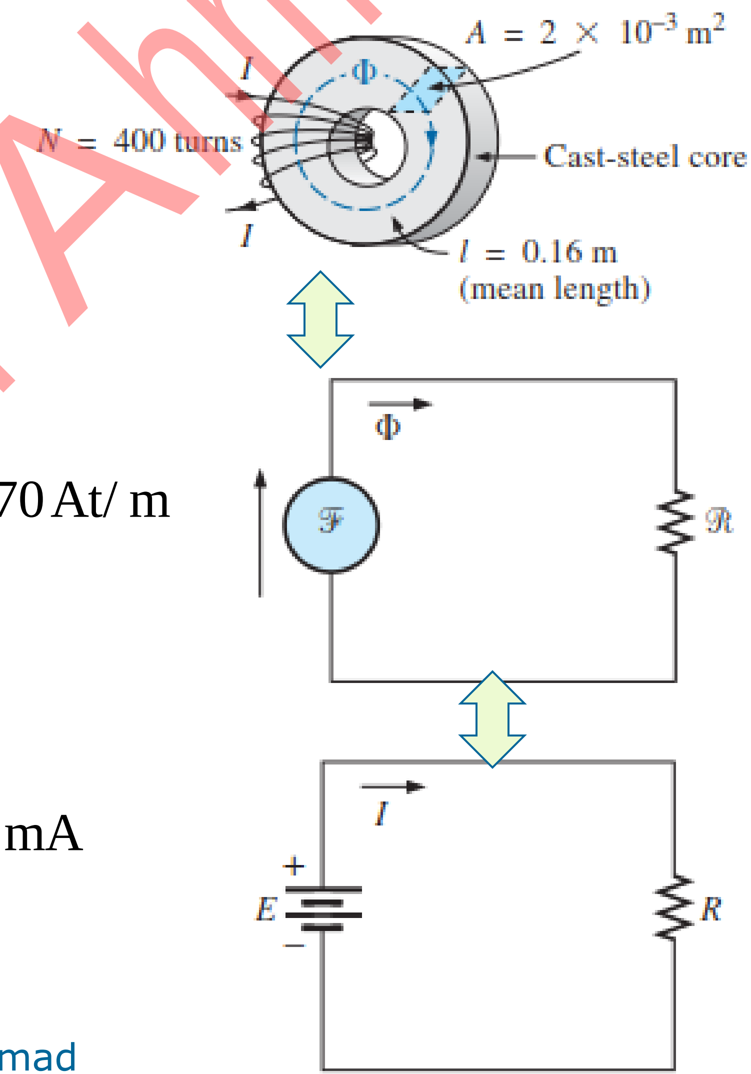
- ϕ is given, and the impressed **mmf** *NI* must be computed – design of motors, generators and transformers
- ***NI*** is given, and the flux ϕ of the magnetic circuit must be found – design of magnetic amplifiers.
- An approach frequently used in the analysis of magnetic circuits is the table method. The columns on the right are reserved for the quantities to be found for each section.

Section	Φ (Wb)	A (m^2)	B (T)	H (At/m)	l (m)	Hl (At)
One continuous section	4×10^{-4}	2×10^{-3}			0.16	

Example 9.7. For the series magnetic circuit in Fig.

a) Find the value of I required to develop a magnetic flux of $\phi = 4\times10^{-4}$ Wb

b) Determine $\boldsymbol{\mu}$ and $\boldsymbol{\mu_r}$ for the material under these conditions.



Solution

a) The flux density *B* is

$$B = \frac{\phi}{A} = \frac{4\times10^{-4}\,\text{Wb}}{2\times10^{-3}\,\text{m}^2} = 2\times10^{-1}\,\text{T} = 0.2\,\text{T}$$

Using the *B*-*H* curves, we can determine the magnetizing force *H*:

$$H(\text{cast steel})\big|_{B=0.2} = 170\,\text{At/m}$$

Applying Ampère's circuital law:

$$NI = H\,l \Rightarrow I = \frac{Hl}{N} = \frac{(170\,\text{At/m})(0.16\,\text{m})}{400\,\text{t}} = 68\,\text{mA}$$

b) The permeability of the material can be found as:

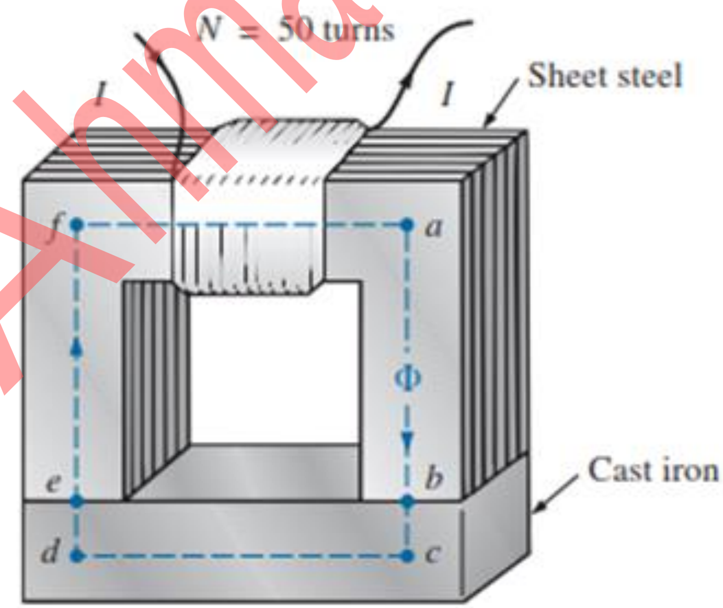
$$\mu = \frac{B}{H} = \frac{0.2\,\text{T}}{170\,\text{At/m}} = 1.176 \times 10^{-3}\ \text{Wb/A} \cdot \text{m}$$

and the relative permeability is

$$\mu_r = \frac{\mu}{\mu_0} = \frac{1.176 \times 10^{-3}\ \text{Wb/A} \cdot \text{m}}{4\pi \times 10^{-7}} = 935.83$$

Section	Φ (Wb)	*A* (m²)	*B* (T)	*H* (At/m)	*l* (m)	*Hl* (At)
One continuous section	4×10^{-4}	2×10^{-3}	**0.2**	**170**	0.16	**27.2**

Example 9.8. The electromagnet in Fig. has picked up a section of cast iron. Determine the current *I* required to establish the indicated flux in the core. Assume that $\phi = 3.5\times10^{-4}\,\text{Wb}, \quad A = 6.452\times10^{-4}\,\text{m}$



$l_{ab} = l_{cd} = l_{ef} = l_{fa} = 0.1016\ \text{m}$

$l_{bc} = l_{de} = 0.0127\ \text{m}$

Solution

$$l_{efab} = 0.1016 + 0.1016 + 0.1016 = 304.8\times10^{-3}\ \text{m}$$

$$l_{bcde} = 0.0127 + 0.1016 + 0.0127 = 127\times10^{-3}\ \text{m}$$

The flux density for each section is

$$B = \frac{\phi}{A} = \frac{3.5\times10^{-4}\ \text{Wb}}{6.452\times10^{-3}\ \text{m}^2} = 0.542\,\text{T}$$

Using the *B*-*H* curves, the magnetizing force is

$$H(\text{sheet steel})\Big|_{B=0.542} \cong 70\ \text{At/m}$$

$$H(\text{cast iron})\Big|_{B=0.542} \cong 1600\ \text{At/m}$$

Determining *Hl* for each section yields

$$H_{efab}\, l_{efab} = (70\,\text{At/ m})(304.8\times10^{-3}\,\text{m}) = 21.34\,\text{At}$$

$$H_{bcde}\, l_{bcde} = (1600\,\text{At/ m})(127\times10^{-3}\,\text{m}) = 203.2\,\text{At}$$

Inserting the above data in Table.

Section	Φ (Wb)	A (m^2)	B (T)	H (At/m)	l (m)	Hl (At)
efab	3.5×10^{-4}	6.452×10^{-4}	0.542	70	304.8×10^{-3}	21.34
bcde	3.5×10^{-4}	6.452×10^{-4}	0.542	1600	127×10^{-3}	203.2

The magnetic circuit equivalent and the electric circuit analogy for the system appear in following Fig.

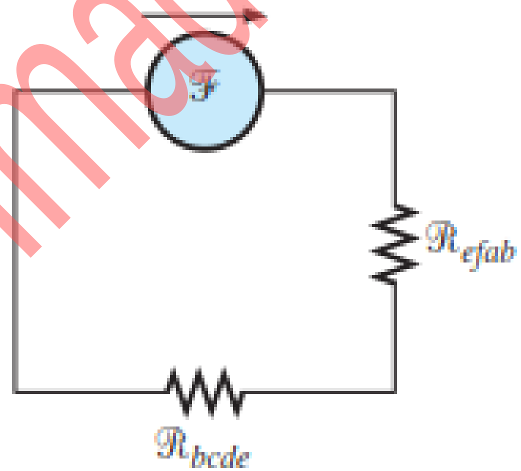
Applying Ampère's circuital law,

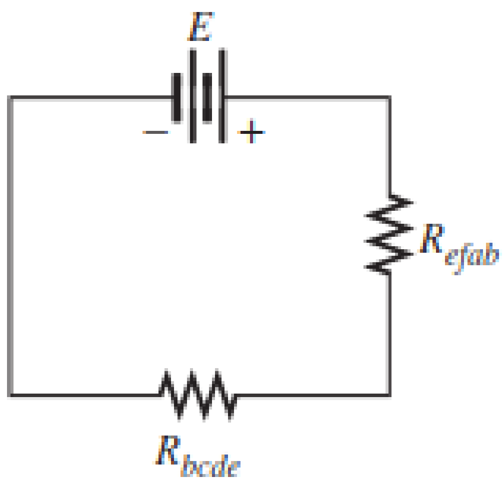
$$NI = H_{efab}\, l_{efab} + H_{bcde} l_{bcde}$$

$$= 21.34\,\text{At} + 203.2\,\text{At} = 224.54\,\text{At}$$

and $\quad (50\,\text{t})I = 224.54\,\text{At}$

$$\Rightarrow I = \frac{224.54\,\text{At}}{50\,\text{t}} = 4.49\,\text{A}$$

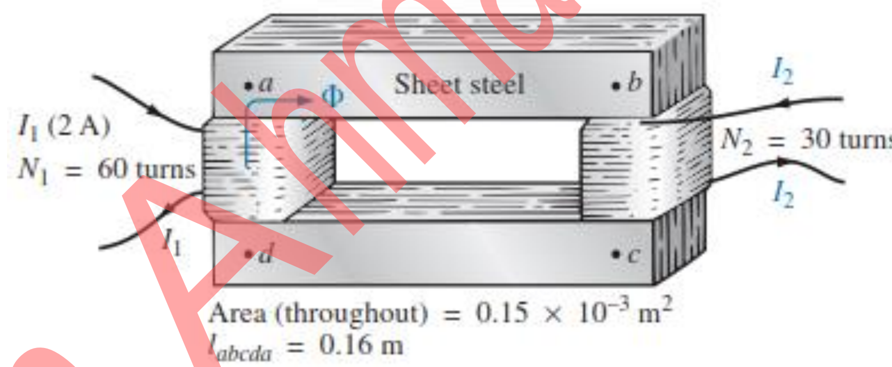




Example 9.9. Determine the secondary current I_2 for the transformer in Fig. if the resultant clockwise flux in the core is $\phi = 1.5\times10^{-5}\,\text{Wb}$

Solution

This is two magnetizing forces. The resulting flux of each is opposing, just as the two sources of voltage are opposing in the electric circuit analogy.

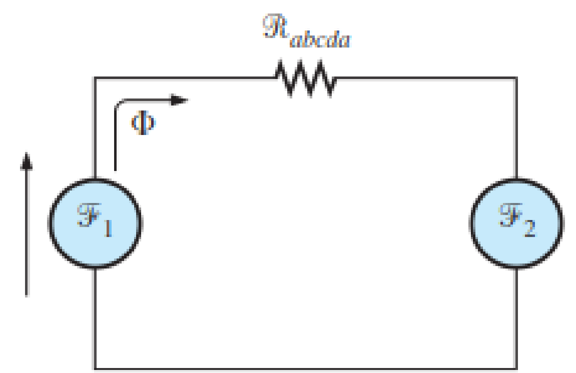


The flux density for each section is

$$B = \frac{\phi}{A} = \frac{1.5\times10^{-5}\,\text{Wb}}{0.15\times10^{-3}\,\text{m}^2} = 0.10\,\text{T}$$

Using the *B-H* curves, the magnetizing force is

$$H\,(\text{sheet steel})\big|_{B=0.10} = 20\,\text{At/m}$$



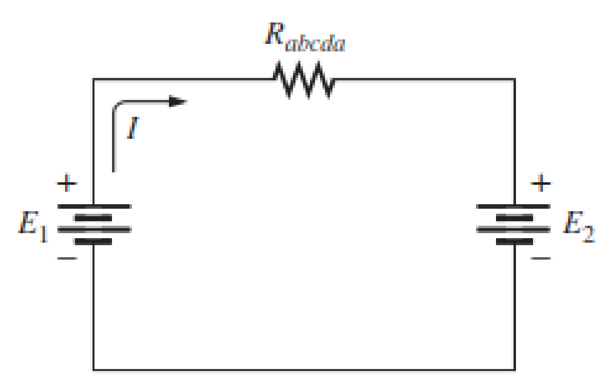
Applying Ampère's circuital law,

$$N_1I_1 - N_2I_2 = N_{abcda}I_{abcda}$$

$$(60\,\text{t})(2\,\text{A}) - (30\,\text{t})(I_2) = (20\,\text{At/m})(0.16\,\text{m})$$

$$120\,\text{At} - (30\,\text{t})(I_2) = 3.2\,\text{At} \Rightarrow (30\,\text{t})(I_2) = 116.8\,\text{At}$$

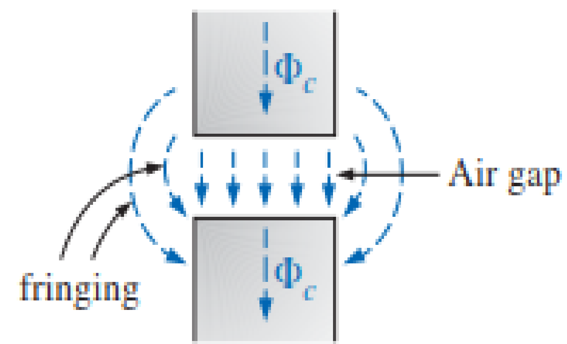
$$\Rightarrow I_2 = \frac{116.8\,\text{At}}{30\,\text{t}} = 3.89\,\text{A}$$



9.9 Air Gaps

Effects of air gaps on a magnetic circuit

- Note the presence of air gaps in the magnetic circuits of the motor and meter in Fig.
- The spreading (انتشار) of the flux lines outside the common area of the core for the air gap in Fig. (a) is known as **fringing (= تهدُّب تدلي)**.
- The flux density of the air gap: $B_g = \frac{\phi_g}{A_g}$

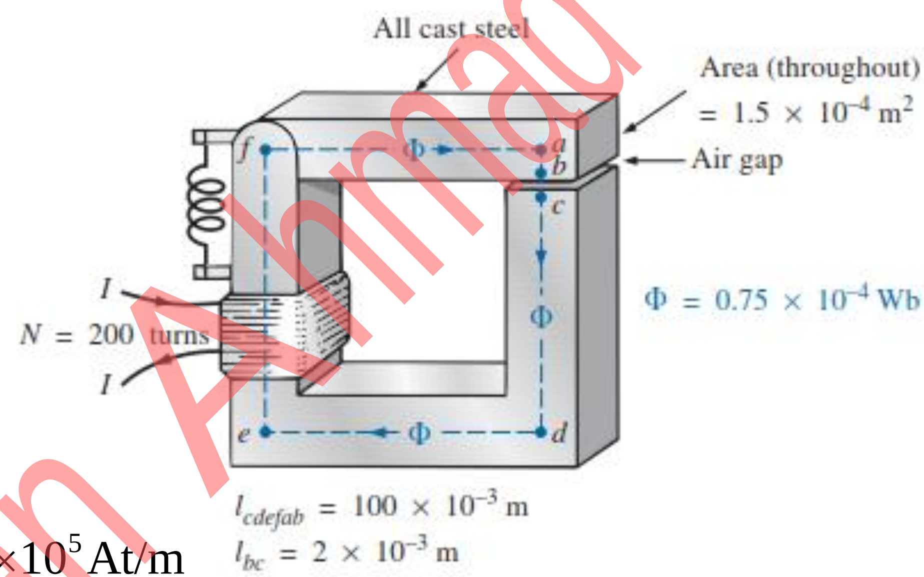


where, $\phi_g = \phi_{core}$, $A_g = A_{core}$

- Assuming the permeability of air is equal to that of free space, the magnetizing force of the air gap is determined by

$$H_g = \frac{B_g}{\mu_0} = \frac{B_g}{4\pi \times 10^{-7}} \Rightarrow H_g = (7.96 \times 10^5) B_g$$

Example 9.10. Find the value of I required to establish a magnetic flux ϕ in the series magnetic circuit in Fig.



Solution

The flux density for each section is

$$B=\frac{\phi}{A}=\frac{0.75\times10^{-4}\,\text{Wb}}{1.5\times10^{-4}\,\text{m}^2}=0.5\text{T}$$

Using the B-H curves, the magnetizing force is $H(\text{cast steel})\big|_{B=0.5}\cong 280\,\text{At/ m}$

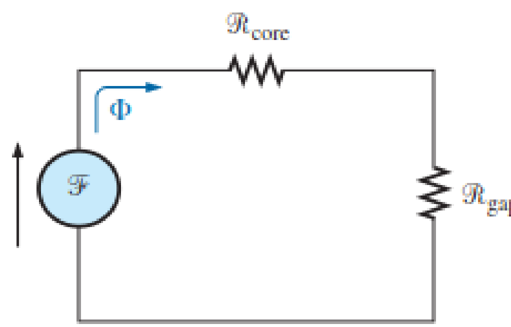
The magnetizing force of the air gap is

$$H_g=(7.96\times10^5)B_g=(7.96\times10^5)(0.5\text{T})=3.98\times10^5\,\text{At/m}$$

The **mmf** drops are

$$H_{\text{core}}I_{\text{core}}=(280\,\text{At/ m})(100\times10^{-3}\,\text{m})=28\,\text{At}$$

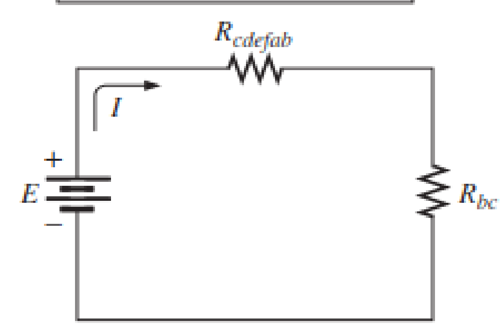
$$H_gI_g=(3.98\,\text{At/ m})(2\times10^{-3}\,\text{m})=796\,\text{At}$$



Applying Ampère's circuital law,

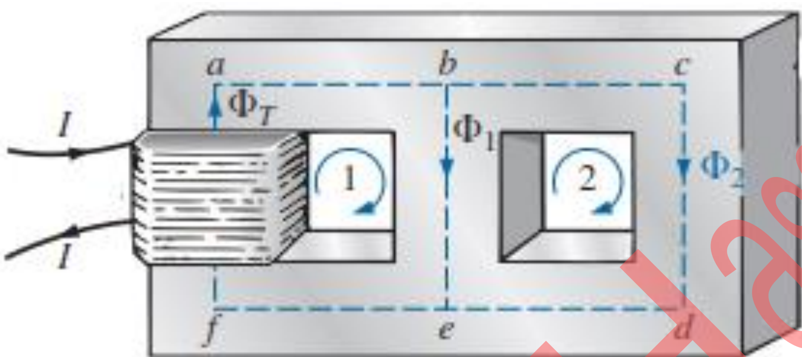
$$NI=H_{core}l_{core}+H_gl_g=28\text{At}+796\text{At}=824\text{At}$$

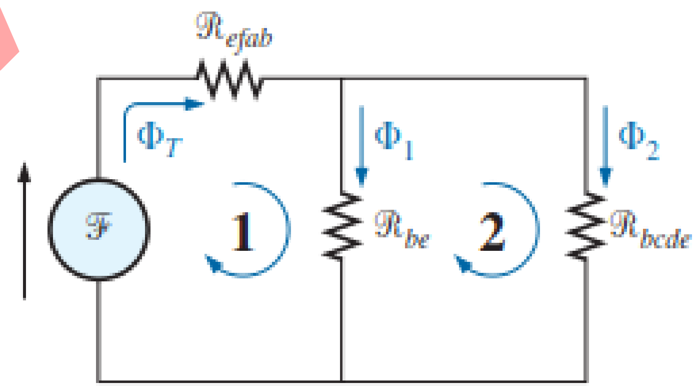
$$\Rightarrow(200\text{t})I=824\text{At}\Rightarrow I=4.12\,\text{A}$$



9.10 Series-Parallel Magnetic Circuits

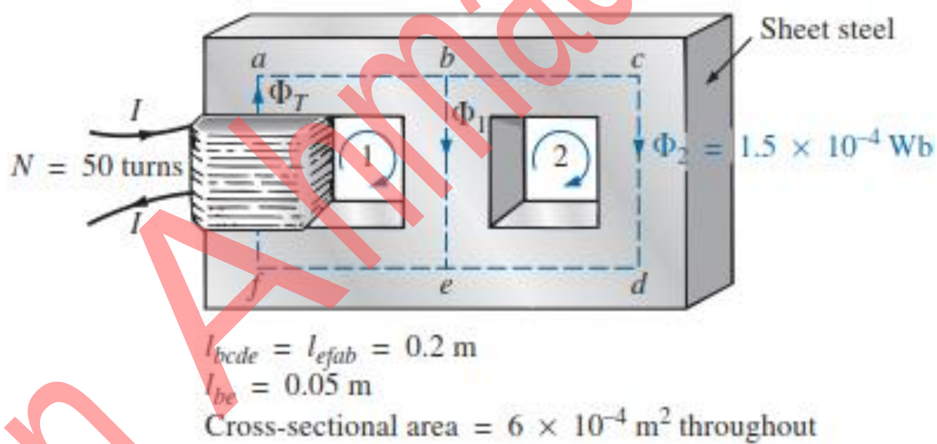
The close analogies between electric and magnetic circuits eventually lead to series-parallel magnetic circuits.





Example 9.11. Determine the current I required to establish a flux of 1.5×10^{-4} Wb in the section of the core indicated in Fig.

Solution



The flux density for 2 section is

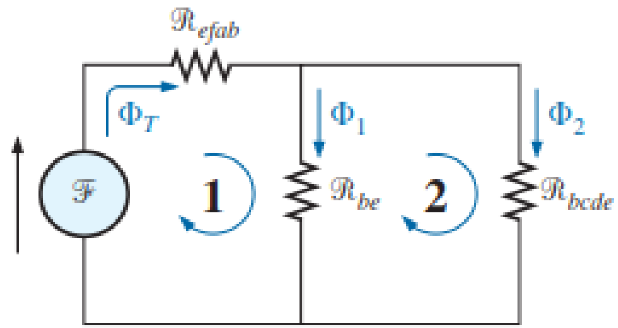
$$B_2 = \frac{\phi_2}{A} = \frac{1.5\times10^{-4}\,\text{Wb}}{6\times10^{-4}\,\text{m}^2} = 0.25\,\text{T}$$

From B-H curves, the magnetizing force is $H_{bcde}\big|_{B=0.25} \cong 40\,\text{At/m}$

Applying Ampère's circuital law around loop 2

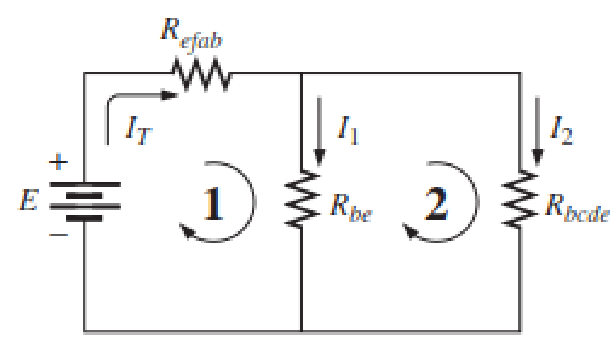
$$\sum F_m = 0 \Rightarrow H_{be}l_{be} - H_{bcde}l_{bcde} = 0$$

$$H_{be}(0.05\,\text{m}) - (40\,\text{At/m})(0.2\,\text{m}) = 0 \Rightarrow H_{be} = \frac{8\,\text{At}}{0.05\,\text{m}} = 160\,\text{At/m}$$



The flux density for 1 section is $B_1\big|_{H=160} \cong 0.97\,\text{T}$

Thus, $\phi_1 = B_1A = (0.97\,\text{T})(6\times10^{-4}\,\text{m}^2) = 5.82\times10^{-4}\,\text{Wb}$



The results for *bcde*, *be*, and *efab* are entered in Table.

Section	Φ (Wb)	A (m²)	B (T)	H (At/m)	l (m)	Hl (At)
bcde	1.5×10^{-4}	6×10^{-4}	0.25	40	0.2	8
be	5.82×10^{-4}	6×10^{-4}	0.97	160	0.05	8
efab		6×10^{-4}			0.2	

From Table and equivalent magnetic circuit, we have

$$\phi_T = \phi_1 + \phi_2 = 5.82\times10^{-4}\,\text{Wb} + 1.5\times10^{-4}\,\text{Wb}$$

$$= 7.32\times10^{-4}\,\text{Wb}$$

$$B = \frac{\phi_T}{A} = \frac{7.32\times10^{-4}\,\text{Wb}}{6\times10^{-4}\,\text{m}^2} = 1.22\,\text{T} \Rightarrow H_{efab}\Big|_{B=1.22\,\text{T}} \cong 400\,\text{At/m}$$

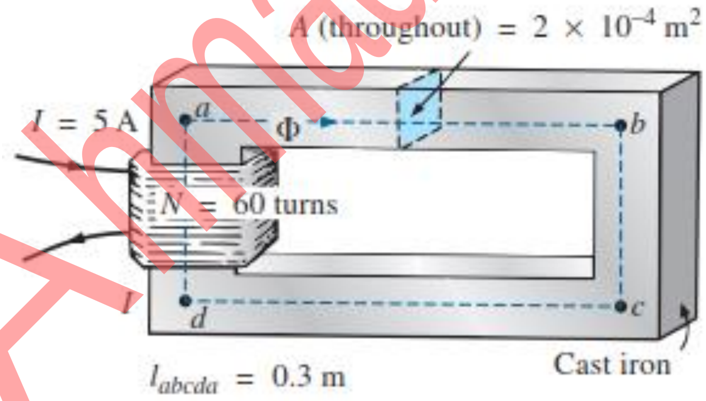
Applying Ampère's circuital law,

$$+NI - H_{efab}l_{efab} - H_{be}l_{be} = 0$$

$$NI = (400\,\text{At/m})(0.2\,\text{m}) + (160\,\text{At/m})(0.05\,\text{m})$$

$$(50\text{t})I = 88\,\text{At} \Rightarrow I = \frac{88\,\text{At}}{50\text{t}} = 1.76\,\text{A}$$

Example 9.12. Calculate the magnetic flux ϕ for the magnetic circuit in Fig.



Solution

By Ampère’s circuital law,

$$NI = H_{abcda} l_{abcda} \Rightarrow H_{abcda} = \frac{NI}{l_{abcda}} = \frac{(60\,\text{t})(5\,\text{A})}{0.3\text{m}}$$

$$= 1000\,\text{At/m}$$

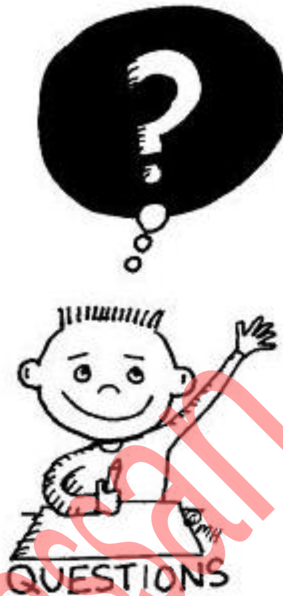
The flux density is $B_{abcda}\big|_{H_{abcda}=1000} \cong 0.39\,\text{T}$

Thus, $\phi = BA = (0.39\,\text{T})(2\times10^{-4}\,\text{m}^2) = 0.78\times10^{-4}\,\text{Wb}$

9.3 Applications

Home work

1. Speakers and Microphones (المايكرفونات ومكبرات الصوت)
2. Hall Effect Sensor (مجسات تأثير هول)
3. Magnetic Reed Switch (switche in alarm systems) (مفتاح قصبي/ ريشة)
4. Magnetic Resonance Imaging (تصوير بالرنين المغناطيسي)
5. dc motors (محركات التيار المستمر)
6. The Solenoid (سولينويد؛ صمام الملف اللولبي)
7. Relays (مبدلات)
8. Meter Movement (عداد الحركة)
9. Magnetic Disk and Tape Read/Write Head (القرص المغناطيسي و شريط القراءة/رأس الكتابة)
10. Magneto-Optical Disk (القرص المغناطيسي البصري).



The end of chapter 9